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THE ART OF PORTFOLIO OPTIMIZATION: MAXIMIZING RETURNS IN THE NIGERIAN STOCK EXCHANGE

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Abstract: Modern Portfolio Theory (MPT), pioneered by Harry Markowitz in the early 1950s, has long been a cornerstone of financial decision-making, particularly in portfolio optimization. Markowitz's mean-variance model aimed to guide investors in selecting assets for their portfolios, determining how to make those selections, and assigning weights to each asset. However, as research has highlighted, the mean-variance approach has its limitations and weaknesses, sparking extensive investigation into its shortcomings.
This paper delves into the focal point of this research, which involves addressing the limitations and assumptions inherent in Markowitz's model. A multitude of scholars, such as Fuerst (2008), Norton (2009), Ceria and Stubbs (2006), Goldfarb and Iyengar (2003), Jorion (1992), Konno and Suzuki (1995), Michaud (1989a), Bowen (1984), Ravipti (2012), and many others, have dedicated their works to thoroughly scrutinizing these shortcomings and restrictions.
Subsequent to the identification of the deficiencies in Markowitz's Mean-Variance model, numerous researchers have sought to enhance and expand the model in various directions. Notable contributions from authors like Jobson, Korkie, and Ratti (1979), Jobson and Korkie (1980), Frost and Savarino (1988), Jorion (1992), Michaud (1998), Polson and Tew (2000), and others have primarily focused on mitigating the estimation error, thus further refining MPT.

Keywords: Modern Portfolio Theory, mean-variance model, portfolio optimization, limitations, estimation error, financial decision-making, asset selection, asset weighting, portfolio diversification.

1. Introduction

In early 1950's, Harry Markowitz designed a financial model otherwise called mean-variance portfolio optimization. This method was designed such that it will help the investors know which asset that will be selected in a portfolio, how the selection will be done and also the weight of each asset in the portfolio. In the paper titled Portfolio selection (1952), Markowitz's outlined the importance of diversification of portfolios. However, research has shown that the Markowitz mean-variance has some weaknesses and a number of limitations. As a matter of fact, the limitations have taken the centre stage of research. Researchers like: Fuerst (2008), Norton (2009), Ceria and Stubbs (2006), Goldfarb and Iyengar (2003), Jorion (1992), Konno and Suzuki

(1995), Michaud (1989a) (1989b), Bowen (1984), Ravipti (2012) etc. discussed the weaknesses, limitations and assumptions in their works.

Since the discovering of the Markowitz's MV limitations and weaknesses, a lot of researchers have been working on the model to improve and develop it in different directions. Authors like Jobson, Korkie and Ratti (1979), Jobson and Korkie (1980), Frost and Savarino (1988), Jorion (1992), Michaud (1998), (1989b), Polson and Tew (2000), etc. worked on the estimation error.

Others like Britten-Jones (2002), Kandel and Stambaugh (1996), Zellner and Chetty (1965), Klein and Bawa (1976) and Brown (1978) worked on the Markowitz's model by using Bayesian approach and predictive probability to improve and develop the model in various ways. Huang (2008) and Markowitz (1993) tried to develop the model to Mean-semi variance. Authors like Galluccio et al. (1998), Laloux et al. (1999), (2008), Bongini et al. (2002), Pafka and Kondor (2002), (2003), Potters et al. (2005), Lindberg (2009) and others brought in Ramdom matrix theory (RMT), which was first proposed and introduced by Wigner (1951) and Laloux and Plerou introduced RMT in financial markets, to improve Markowitz's portfolio optimisation.

In this paper, we aim to optimize a portfolio containing stocks from the financial services sector of the Nigerian stock Exchange (NSE) using Markowitz's portfolio selection model and a three-objective linear programming model to allocate different percentage of weight to different assets to obtain an optimal/feasible portfolio, diversification of assets, and later we brought in cross - correlation of the individual stock of the sector to show the relationship between any two assets chosen in the correlation matrix. The rest of the paper is organised as follows. In section 2, we describe the nature of the empirical data used in the analysis. In section 3, we present the methodologies, theoretical background on mean-variance optimization; expected return and risk of the portfolio of the assets, constrain objective programme and cross - correlation of assets. Section 4, shows and discusses the main empirical result. Finally, section 5 concludes the paper.

2. Data

We obtained our data from NSE,which is made up of eleven (11) sectors, but our analysis is on the financial services. The financial services are about 57assets from the time our analysis started. But we have to bring them down to 24 assets only in the course of our study. This development was necessary because some of assets were delisted from NSE due to some banks merging together and some bigger ones acquiring smaller one after the global melt down in order to meet up with the new capital base for the financial institutions operating in the country as ordered by the Central Bank of Nigeria(CBN). Also, some of the company under this sectordid not trade more regularly within the time interval of our analysis and therefore, was removed. The data set used is the daily closing price of the stock data listed in the financial services of NSE. We have 1485 daily closing prices running from 3rd August 2009 to 4th August 2015, excluding weekends and public holidays in Nigeria (Nationwide). These stock price data were converted into 1484 logarithmic returns and was used in our analysis. Let ( ) be the closing price of the index on day ( ) of stock and define the natural logarithmic returns of the index (i.e. the log-difference of (+ 1) ( )) as

( ) = (+ 1) − ( ) (1)

Where ( ) has 1484 observation? Before establishing the portfolio selection process, we compute the mean return and standard deviation of each stock .

The table below shows the mean and standard variation of the individual stocks.

3. Theoretical background and methodology

3.1 Expected return of a portfolio.

The portfolio of n assets has each asset delivers a return of ( ) at the time . Each ( ) has its mean and variance which is donated as ( ) ( ) respectively. The money invested in the assets is regarded as weight of the asset ( )(which is less than 1 and sometimes a negative number is allowed if there is a short selling of any asset).Therefore, the summation of the individual weights of the assets that form the portfolio is 1, thus,∑ = 1 and it is obvious to see that

= [ ] =

Therefore,

= (2)

and

( ) = [(−)] = (−)

Which implies that

( ) = , (3)

,

…

Which is written as ( ) = , where = ⋮ and = ⋮ ⋱ ⋮ are called the weight

⋯ vector and covariance matrix respectively.

Let's recall that the correlation between any two assets is

= ___ (4)

Where , are the standard deviation of and respectively, while is the coefficient of correlation of and , for , = 1, 2, … , . The coefficient of correlation plays a great role in the portfolio diversification, if well managed; the coefficient of correlation will reduce the risk to a bearable level. In other words, the risk of a portfolio decreases as the coefficient of correlation of the assets moves from positive to negative.

Table 1

	ACCESS	AICO	CONTINSURE	CORNERST	CUSTODYINS	DIAMONDBNK	FBNH	FCMB	FIDELITYBK	GUARANTY	MANSARD	NEM	NIGERINS	PRESTIGE	ROYALEX	SKYEBANK	STERLNBANK	TRANSCORP	UAC-PROP	UBA	UBN	WAPIC	WEMABANK	ZENITHBANK
Average	-0.0218%	-0.0229%	-0.0284%	-0.0334%	0.0253%	-0.0510%	-0.0606%	-0.0640%	-0.0409%	0.0335%	-0.0140%	0.4518%	-0.0736%	-0.1566%	-0.0447%	0.1844%	0.0235%	0.0987%	-0.0235%	0.0081%	-0.1683%	0.0047%	0.0182%	0.0058 %
Variance	0.0686%	0.1036%	0.0878%	0.0338%	0.0971%	0.0785%	0.0546%	0.0692%	0.0779%	0.0577%	0.0859%	3.1963%	0.0416%	0.0824%	0.0677%	1.1029%	0.0960%	0.1124%	0.0836%	0.2022%	0.5122%	0.2601%	0.2148%	0.0605 %

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Standard D	2.6188%	3.2192%	2.9637%	1.8390%	3.1164%	2.8021%	2.3362%	2.6303%	2.7915%	2.4023%	2.9312%	17.8781%	2.0384%	2.8703%	2.6021%	10.5018%	3.0976%	3.3522%	2.8906%	4.4964%	7.1569%	5.0997%	4.6348%	2.4606%

Table showing individual assets return, variance and risksfrom financial sector of NSE It is understood that the intention of every investor is to make as much gain as possible; therefore, it will be his wish to select the optimal portfolio which will maximize his expected return.
This can be expressed in a mathematical form thus;

( ) = [ ]

. = 1 (5)

≥ 0, = 1, 2, … ,

This implies that all the funds will be invested in the *n* assets and in the course of our analysis there will be no short selling.
An investor wishes to build a feasible portfolio*; this feasible portfolio becomes the efficient one if it satisfies the following condition with at least one strict inequality;

1. ( ) ≤ (*)
2. ( ) ≥ (*)
3. ( ) ≤ (*)

Where ( ), ( ) ( ) are the expected return, risk and the sharpe ratio of the portfolio =
(, , … ,).
This gives us a model of three - objective programming problem which shows that the expected return and the sharpe ratio will be maximised and the variance will be minimised.
Thus the model becomes,
( )
(, , … ,) ( ) (6)

= 1
With ≥ 0, = 1, 2, … , .

The mean-variance criterion is also equivalent to the expected utility approach for any risk-averse utility function, when all returns are normal random variables. Since the probability distribution is defined on mean and standard deviation, it implies that expected utility is a function of mean and standard deviation. When the utility is risk averse, therefore,

[( )] = (,)(7)
with __ > 0 and __ < 0.

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Where *U* is the utility function, is the mean and is the standard deviation To solve the maximization function problem of a portfolio is therefore, a linear combination of assets that are normal random variables with respect to all feasible combinations. Our task now is to find that will maximize (,) with respect to all feasible combination

4. Empirical results and its analysis

Our main aim is to maximize the expected return and minimise the variance of the expected return of the portfolio containing assets from the financial services using the daily closing prices of the assets from 3rd of August 2009 to 4th of August 2015. This becomes 1484 days when all weekends and public holidays in Nigeria are excluded.

4.1 Portfolio1 Equally weighted Portfolio

We first constructed a portfolio that is equally weighted using the daily closing prices of the market, we got a portfolio which the return is 0.00162% and the standard deviation is 1.28% (see Table 2 and 3). Though, the standard deviation of the portfolio seems to be better than what we have from the market (see Table 1 and 4), but the return is very poor. In Table4 and figure 1, we can see that the single asset with the least risk is CORNERST, which is 1.84% but unfortunately, with a return that is very poor. Now our objective is to maximise the portfolio's return with a portfolio standard deviation which should be less than or equal to the least risk, (in other words we want construct a portfolio that the standard deviation will be less than or equal to that of CORNERST but the return will be above its return).

4.1 Portfolio 2: Maximization of the return

Therefore, we apply

$$(\quad , \quad , \dots , \quad) = \quad - \qquad (8)$$

Subject:

$$(\quad , \quad , \dots , \quad) = \frac{1}{-1} \quad , \leq 1.84\%$$

$$(\quad , \quad , \dots , \quad) = \quad - 1 = 0$$

Where is the weight of individual assets, n is the number of the observations. After the simulation, the weights were distributed among the assets but assets like UBA, UBN, Diamond bank, ACCESS, FBNH, Fidelity bank, FCMB etc. were allocated with 0% of the weight while assets like Transcorp, Guaranty trust bank and Custody were given more percentage of the weight (see Table 4).

Table 2

		Portfolios		
	Equal Wt	Max Return	Min St Dev	Max SR
	None	at σ ≤	at μ =	None
Value of Constr	N/a	1.840%	0.450%	N/a
ACCESS	4.1666%	0.0000%	0.0000%	0.0000 %
AIICO	4.1666%	0.0000%	0.0000%	0.0000 %

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CONTINSURE	4.1666%	0.0000%	0.0000%	0.0000 %
CORNERST	4.1666%	0.0000%	0.0000%	0.0000 %
CUSTODYINS	4.1666%	11.9229%	0.0000%	10.3272 %
DIAMONDBNK	4.1666%	0.0000%	0.0000%	0.0000 %
FBNH	4.1666%	0.0000%	0.0000%	0.0000 %
FCMB	4.1666%	0.0000%	0.0000%	0.0000 %
FIDELITYBK	4.1666%	0.0000%	0.0000%	0.0000 %
GUARANTY	4.1666%	27.2599%	0.0000%	26.4968 %
MANSARD	4.1666%	0.0000%	0.0000%	0.0000 %
NEM	4.1666%	5.1779%	100.0000%	6.3839 %
NIGERINS	4.1666%	0.0000%	0.0000%	0.0000 %
PRESTIGE	4.1666%	0.0000%	0.0000%	0.0000 %
ROYALEX	4.1666%	0.0000%	0.0000%	0.0000 %
SKYEBANK	4.1666%	5.6436%	0.0000%	6.7855 %
STERLNBANK	4.1666%	8.5136%	0.0000%	6.7090 %
TRANSCORP	4.1666%	36.2113%	0.0000%	40.8552 %
UAC-PROP	4.1666%	0.0000%	0.0000%	0.0000 %
UBA	4.1666%	0.0000%	0.0000%	0.0000 %
UBN	4.1666%	0.0000%	0.0000%	0.0000 %
WAPIC	4.1666%	2.0102%	0.0000%	0.7009 %
WEMABANK	4.1666%	2.9119%	0.0000%	1.7415 %

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ZENITHBANK	4.1666%	0.3488%	0.0000%	0.0000 %
Σ w	100.00%	100.00%	100.00%	100.00 %
μ_p	0.00162%	0.0839%	0.450%	0.0946 %
σ_p	1.281%	1.840%	17.89%	2.060 %
μ/σ	0.126%	4.56%	2.52%	4.59%

Table of our four different portfolios constructed.

Though in this new portfolio, we got a standard deviation that is greater than that of the portfolio with equal weighted assets, but the return is very encouraging. The return is about 52 times of the return of the said portfolio (see Table 4). Again, if we look at the return of the asset with the least standard deviation (CORNERST with σ = 1.84%, see Table 3), you will notice that it cannot be compared to our new return. Finally, if we look at the Sharpe ration (SR) of the portfolios, SR of the equal weighted portfolio and our new portfolio are 0.12% and 4.56% respectively (see Table 4) and the stock with the least standard deviation has its SR to be 1.81% (Table 2), this shows that 4.56% is best among all.

4.3 Portfolio 3: Minimization of Standard Deviation

4.4

In this case, we want to minimise the standard deviation of the single asset with maximum SD (NEM) which is 17.88% (Table 1), to see if we will get a lower SD and an improved return (which may not necessarily be equal to the return of the said asset). Therefore, we apply

$$(\quad , \quad , \ldots , \quad) = \frac{1}{\quad - 1} \qquad , \tag{9}$$

Subject to

$$(\quad , \quad , \ldots , \quad) = \quad - \quad \geq 0.450\%$$

$$(\quad , \quad , \ldots , \quad) = \quad - 1 = 0$$

After the simulation, we got a funny result where 100% of our weight is allocated to NEM, with return and SD equal to what we had abinitio and therefore this portfolio is not acceptable.

4.5 Portfolio 4: Maximization of Sharpe ratio

Finally, we maximise the sharpe ratio SR. Here we have the equation as follows

$$(\quad , \quad , \ldots , \quad) = \tag{10}$$

$$(\quad , \quad , \ldots , \quad) = \quad - 1 = 0$$

Again, we have the return to be 0.095%, the SD to be 2.06% and SR 4.6%. The weights were loaded in Transcorp, Guaranty trust bank and Custody assets with very few distributed among Skye, Sterling and Wema Banks, others are Wapic and NEM.

Comparison of the results, that is, equally weighted portfolio, Max. Return, Min. Standard deviation and Max

SR as shown in Table 3

		Portfolios		
	Equal Wt	Max Return	Min St Dev	Max SR
μ_p	0.00162%	0.084%	0.45%	0.095 %
σ_p	1.28%	1.84%	17.89%	2.06 %
μ/σ	0.13%	4.56%	2.52%	4.60%

Table 3.A table showing the return, risk and sharpe ratio of the four portfolios constructed.
If we take the equally weighted portfolio as our pivotal portfolio, with return, standard deviation and sharpe ratio as 0.00162%, 1.28% and 0.13% respectively, we notice that it return was below expectations. Though the risk is very minimal but the return and the sharpe ratio show that it is not a good idea to invest in the sector with an equally weighted portfolio. The portfolio that minimizes standard deviation has the highest return but the risk is too much and the sharpe ratio is not encouraging, also the simulation allocated 100% of the weight to one stock (NEM) which does not encourage diversification of funds. Therefore, these make it not healthy for investment. We are now left with two options which are, Max Return and Max SR which have their returns as multiples of 52 and 59 of the return of the equally weighted portfolio respectively. Though the risk value of both is greater than the value of the equally weighted portfolio but the sharpe ratios are better, which again are multiples of 46 on approximate of the equally weighted portfolio.

Table 4.

Individual	Assets			
	Average	Variance	Standard D	μ/σ
ACCESS	-0.022%	0.069%	2.620%	-0.832557 %
AIICO	-0.023%	0.100%	3.220%	-0.711739 %
CONTINSURE	-0.028%	0.088%	2.960%	-0.960405 %
CORNERST	-0.033%	0.034%	1.840%	-1.812935 %
CUSTODYINS	0.025%	0.097%	3.120%	0.809808 %
DIAMONDBNK	-0.051%	0.079%	2.800%	-1.821179 %
FBNH	-0.061%	0.055%	2.340%	-2.589444 %
FCMB	-0.064%	0.069%	2.630%	-2.431863 %

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FIDELITYBK	-0.041%	0.078%	2.790%	-1.464946%
GUARANTY	0.033%	0.058%	2.400%	1.394083%
MANSARD	-0.014%	0.086%	2.930%	-0.478976%
NEM	0.450%	3.200%	17.880%	2.516779%
NIGERINS	-0.074%	0.042%	2.040%	-3.609265%
PRESTIGE	-0.160%	0.082%	2.870%	-5.574913%
ROYALEX	-0.045%	0.068%	2.600%	-1.718692%
SKYEBANK	0.180%	1.100%	10.500%	1.714286%
STERLNBANK	0.024%	0.096%	3.100%	0.758484%
TRANSCORP	0.099%	0.110%	3.350%	2.945343%
UAC-PROP	-0.023%	0.084%	2.890%	-0.813114%
UBA	0.008%	0.200%	4.500%	0.181098%
UBN	-0.170%	0.510%	7.160%	-2.374302%
WAPIC	0.005%	0.260%	5.100%	0.092069%
WEMABANK	0.018%	0.210%	4.630%	0.392613%
ZENITHBANK	0.006%	0.061%	2.460%	0.237573%

Table showing individual assets return, risks and sharpe ratio from financial sector of NSE
Fig.1

Histogram showing the return, risk and sharpe ratio of the constructed portfolios.
Fig 2.

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	ACCESS	AIICO	CONTINSU	CORNERST	CUSTODYI	DIAMOND	FBNH	FCMB	FIDELITYB	KGUARANT	MANSARD	NEM	NIGERINS	PRESTIGE	ROYALEX	SKYEBANK	STERLNBA	TRANSCO	RUACPRO	UBA	UBN	WAPIC	WEMABA	NZENITHBAN
ACCESS	1	0.15076	0.056041	-0.01129	0.054676	0.239825	0.195369	0.127066	0.187493	0.165777	0.024933	0.022425	0.017909	0.013768	0.062961	-0.00181	0.12783	-0.02529	0.030219	0.067869	0.002778	0.053063	0.016861	0.223521
AIICO	0.15076	1	0.031982	0.009987	0.024212	0.067551	0.110694	0.061034	0.043642	0.132721	-0.02568	0.006563	0.015028	0.000717	0.006873	-0.00154	0.082536	-0.01036	0.002989	0.022636	-0.04742	0.018642	0.045983	0.120533
CONTINSU	0.056041	0.031982	1	0.061441	-0.07555	0.013918	0.044602	-0.00133	0.07853	0.016289	0.020515	0.010001	0.017147	-0.04602	-0.02966	0.011193	0.003815	0.050338	-0.00212	0.036778	0.007538	0.010738	-0.01125	0.031048
CORNERS	T -0.01129	0.009987	0.061441	1	0.01274	0.005558	0.008376	0.057548	0.01488	-0.02078	-0.02035	0.006918	0.015995	-0.02565	-0.02834	0.013671	0.032636	0.006602	0.058526	-0.01141	0.001812	0.038141	0.024661	-0.02996
CUSTODYI	0.054676	0.024212	-0.07555	0.01274	1	0.049431	0.040057	0.045621	0.037637	0.046467	-0.04119	-0.00237	0.014387	0.007517	-0.00912	0.007332	0.072169	-0.00514	0.0397	0.029588	0.01162	-0.0102	0.018732	0.053487
DIAMOND	0.239825	0.067551	0.013918	0.005558	0.049431	1	0.174686	0.097734	0.290397	0.146793	-0.02485	0.000141	-0.0202	0.061032	0.031771	0.065686	0.142097	0.001916	0.01464	0.15934	-0.00886	0.102077	0.059745	0.145028
FBNH	0.195369	0.110694	0.044602	0.008376	0.040057	0.174686	1	0.117732	0.142378	0.372794	0.028788	0.005897	0.001805	-0.07033	0.013712	-0.01355	0.118883	-0.03332	0.080751	0.091059	-0.0229	0.020163	0.039201	0.385486
FCMB	0.127066	0.061034	-0.00133	0.057548	0.045621	0.097734	0.117732	1	0.106594	0.112151	0.029501	0.009275	-0.00342	-0.02713	0.006427	0.035481	0.164312	0.059917	0.072263	0.075343	-0.04152	0.0034	-0.00382	0.118524
FIDELITYB	K 0.187493	0.043642	0.07853	0.01488	0.037637	0.290397	0.142378	0.106594	1	0.140685	0.021668	-0.00039	0.013386	0.045131	0.088691	0.055757	0.117026	0.006328	0.015783	0.097524	0.013021	0.080512	0.073797	0.222817
GUARANT	0.165777	0.132721	0.016289	-0.02078	0.046467	0.146793	0.372794	0.112151	0.140685	1	0.008122	0.000976	-0.02638	-0.00331	0.01757	0.005144	0.108429	-0.04327	0.068898	0.075798	0.005193	0.053239	0.028267	0.357704

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MANSARD	0.024933	-0.02568	0.020515	-0.02035	-0.04119	-0.02485	0.028788	0.029501	0.021668	0.008122	1	0.007896	0.042376	0.002973	0.051473	0.000686	-0.00373	0.016646	-0.02154	0.027622	0.043367	0.007491	0.042028	0.06349
NEM	0.022425	0.006563	0.010001	0.006918	-0.00237	0.00141	0.005897	0.009275	-0.00039	0.000976	0.007896	1	0.010726	-0.00539	0.008879	0.00444	0.004899	-0.01398	-0.00852	0.034025	-0.00015	-0.00128	0.007008	-0.00711
NIGERINS	0.017909	0.015028	0.017147	0.015995	0.014387	-0.0202	0.001805	-0.00342	0.013386	-0.02638	0.042376	0.010726	1	0.044131	-0.02136	0.001065	-0.00182	-0.01568	0.014461	0.015311	0.019744	0.049056	0.043319	0.018145
PRESTIGE	0.013768	0.000717	-0.04602	-0.02565	0.007517	0.06103	-0.07033	-0.02713	0.045131	-0.00331	0.002973	-0.00539	0.044131	1	0.021425	0.002879	-0.00308	-0.01809	-0.03582	-0.02624	-0.02189	0.014432	-0.01486	0.015711
ROYALEX	0.062961	0.006873	-0.02966	-0.02834	-0.00912	0.031771	0.013712	0.006427	0.088691	0.01757	0.051473	0.008879	-0.02136	0.021425	1	0.012932	0.067454	0.018112	-0.01278	0.002731	0.002697	0.011582	0.00495	0.038326
SKYEBANK	-0.00181	-0.00154	0.011193	0.013671	0.007332	0.065686	-0.01355	0.035481	0.055757	0.005144	0.00686	0.00444	0.01065	0.002879	0.012932	1	0.025861	0.029057	0.024996	0.753359	-0.01009	-0.0022	0.019219	0.019501
STERLNBA	0.12783	0.082536	0.003815	0.032636	0.072169	0.142097	0.118883	0.164312	0.117026	0.108429	-0.00373	0.004899	-0.00182	-0.00308	0.067454	0.025861	1	0.006659	0.070407	0.03845	0.008223	0.006269	0.074596	0.128894
TRANSCO	R -0.02529	-0.01036	0.050338	0.006602	-0.00514	0.01916	-0.03332	0.059917	0.006328	-0.04327	0.016646	-0.01398	-0.01568	-0.01809	0.018112	0.029057	0.006659	1	0.022529	0.104056	0.000208	-0.01773	0.02563	-0.05526
UACPRO	0.030219	0.002989	-0.00212	0.058526	0.0397	0.01464	0.080751	0.072263	0.015783	0.068898	-0.02154	-0.00852	0.014461	-0.03582	-0.01278	0.024996	0.070407	0.022529	1	0.03557	-0.02204	0.057531	0.02092	0.02097
UBA	0.067869	0.022636	0.036778	-0.01141	0.029588	0.15934	0.091059	0.075343	0.097524	0.075798	0.027622	0.034025	0.015311	-0.02624	0.002731	0.753359	0.03845	0.104056	0.03557	1	-0.02388	0.059057	0.034955	0.09979
UBN	0.002778	-0.04742	0.007538	0.001812	0.01162	-0.00886	-0.0229	-0.04152	0.013021	0.005193	0.043367	-0.00015	0.019744	-0.02189	0.002697	-0.01009	0.008223	0.000208	-0.02204	-0.02388	1	0.004097	-0.00339	-0.00381

WAPIC	0.053063	0.018642	0.010738	0.038141	-0.0102	0.102077	0.020163	0.0034	0.080512	0.053239	0.007491	-0.00128	0.049056	0.014432	0.011582	-0.0022	0.006269	-0.01773	0.057531	0.059057	0.004097	1	0.012808	0.069573
WEMABA	N0.016861	0.045983	-0.01125	0.024661	0.018732	0.059745	0.039201	-0.00382	0.073797	0.028267	0.042028	0.007008	0.043319	-0.01486	0.000495	0.019219	0.074596	0.02563	0.02092	0.034955	-0.00339	0.012808	1	0.073013
ZENITHBA	0.223521	0.120533	0.031048	-0.02996	0.053487	0.145028	0.385486	0.118524	0.222817	0.357704	0.06349	-0.00711	0.018145	0.015711	0.038326	0.019501	0.128894	-0.05526	0.02097	0.09979	-0.00381	0.069573	0.073013	1

The correlation matrix of the assets in the financial sector of NSE

5. Conclusion

We were able to construct four portfolios as we can see the summery in Table 3. The first one is equally weighted and the return is so small that an investor who wants a profit will not be advised to invest in such portfolio. Secondly, the portfolio that was formed with equation (9) gave the highest return with a very high standard deviation which is not encouraging. Besides, the idea of diversification was killed because the whole fund was allocated to one stock which is NSE. Therefore we advise investors to disregard this. Finally, the equations (8) and (10) gave us something closer to what we want, an appreciable return and a risk that can be tolerated and above all, their sharpe ratios are within acceptable boundaries when compared with the former two. Investors who want to invest in this sector are advised to invest in the portfolio of equation (10), which we consider to be the optimal portfolio. Though, the risk is slightly above the other but the return and the sharpe ratio are very encouraging. Furthermore, we can see from the interaction of the stocks in the correlation matrix (fig 2), that the assets selected in the portfolio move in such direction that will reduces risk. Investors are highly advised to invest in this portfolio.

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