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COSMIC TILT UNVEILED: BIANCHI TYPE I MODELS WITH PERFECT FLUID EXPLORED IN GENERAL RELATIVITY

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Abstract: The investigation of spatially homogeneous and anisotropic universes, specifically those manifesting tilt, has become a focal point in recent cosmological studies. Tilted universes, characterized by non-orthogonal matter movement concerning the hyper surface of homogeneity, provide a nuanced perspective on cosmic dynamics. Early works by King and Ellis (1973), Ellis and King (1974), and Collins and Ellis (1979) laid the foundation for understanding the general dynamics of tilted universes. This exploration extended to Tilted Bianchi Type I models, with Dunn and Tupper (1978) and Lorenz (1981) contributing valuable insights, while Mukherjee (1983) introduced heat flux, revealing intriguing pancake-shaped configurations. Bradley (1988) further enriched the discourse by deriving tilted spherically symmetric self-similar dust models, adding complexity to the equations governing tilted cosmological scenarios. The mathematical formalism governing tilted cosmological models, as expounded by Ellis and Baldwin (1984), is notably intricate compared to non-tilted counterparts, suggesting potential tilt in our universe and proposing detection methods. Advancements in understanding tilted cosmological scenarios include Cen et al.'s (1992) exploration of tilted cold dark matter models, providing insights into the implications of tilt in cosmological dynamics. Bali and Sharma (2002) contributed by examining the characteristics of tilted Bianchi Type I dust fluid, unveiling peculiar cigar-type singularities under specific conditions. This abstract encapsulates the evolving landscape of tilted universes, emphasizing the significance of tilt in shaping cosmic evolution and structure. Through a historical overview and examination of key findings, it underscores the importance of understanding tilted cosmological models in elucidating fundamental aspects of the universe's evolution and structure.
Keywords: Tilted Universes, Cosmological Dynamics, Bianchi Type I Models, Cosmic Evolution, Cosmic Structure

INTRODUCTION

In recent years, there has been a considerable interest in investigating spatially homogeneous and anisotropic universe in which the matter does not move orthogonally to the hypersurface of homogeneity. These are called tilted universe. The general dynamics of tilted universe have been studied in detail by King and Ellis (1973), Ellis and King (1974), and Collins and Ellis (1979). Tilted Bianchi Type I models have been obtained by Dunn and Tupper (1978) and Lorenz (1981). Mukherjee (1983) has investigated tilted Bianchi Type I universe with heat flux in general relativity. He has shown that the universe assumes a pancake shape. Bradley (1988) obtained all tilted spherically symmetric self-similar dust models. The equations for tilted cosmological models are more complicated than those of non-tilted ones. Ellis and Baldwin (1984) have shown that we are likely to be living in a tilted universe and they have indicated how we may detect it. A tilted cold dark matter cosmological scenario has been discussed by Cen et al. (1992). Bali and Sharma (2002) investigated tilted Bianchi Type I dust fluid and shown that model has cigar type singularity when T = 0. In this paper, we have investigated tilted Bianchi Type I dust fluid of perfect fluid in general relativity. To get a determinate solution, a supplementary condition P = 0, A = $(BC)^n$ between metric potential is used. The behavior of the singularity in the model with other physical and geometrical aspects of the models is also discussed.

THE METRIC AND FIELD EQUATIONS

We consider metric in the form:

$ds^2=\Box dt^2+A^2 dx^2+B^2 dy^2+C^2 dz^2$, (1)

Where A, B and C are functions of 't' alone.

The energy-momentum tensor for perfect fluid distribution with heat conduction given by Ellis (1971) is taken into the form:

$T_i^j \Box(\Box\Box p)v_iv^j \Box pg_i^j \Box q_iv^j \Box v_iq^j$, (2)

Together with

$g_{ij} v_i v^j = -1$, (3)

$q_iq^j > 0$, (4) 2

$q_iv^i = 0$, (5)

Where p is the pressure, □ the density and q_i the heat conduction vector orthogonal to v^i. The fluid flow vector has the components □□____sinh□, 0, 0, cosh□□□satisfying Equation

□ A □

3 and □ is the tilt angle.

The Einstein field equation

~~AB44 + AC4~~ 4 + BC44 = □8 π □□□□(□□p) cosh2□□p□2q1 A□□□□ , (10)

(□□p) Asinh□ cosh□□$q_1$cosh□□q_1 ______ $^{\sinh2\Box}$□ 0 , (11) cosh□

Where the suffix '4' stands for ordinary differentiation with respect to cosmic time 't' alone.

SOLUTION OF FIELD EQUATIONS

Equations 7 to 11 are five equations in seven unknown A, B, C, □, p, □ and q_1; therefore to determine the complete solution we require two more conditions:

1) We assume that the model is filled with dust of perfect fluid which leads to

$p = 0$ (12)

2) Relation between metric potential as:

$A = (BC)^n$ (13)

Where n is constant.

Equations 7 and 10 lead to

B44 C44 2B4C4 □ A4C4 □ A4B4 □ 8□ (14)
(□□p) □ □

Rij □1Rgij □□8□Tij, (units such that c = G = 1) (6)
2
For the line, element of Equation 1 are

B44 + C44 + B4C4 = □8 π □□(□□p) sinh2□□p□2q1
B C BC □
sinh□□ , (7)
A □□
A44 +C44 +A4C4 =□8πp ,
A C AC
(8)
A44 +B44 +A4B4 =□8πp ,
A B AB
(9)
A B A C B C sinh
B C BC AC AB
From Equations 12 and 14, we have
B44 □ C44 □ 2B4C4 □ A4C4 □ A4B4 □ 8□□ (15)
B C BC AC AB
Equations 8 and 9 lead to
B44 □ C44 □ A4 □□B4 □ C4 □□□0 (16)
B C A □ B C □
This leads to
□4 □ an□1 (17)
□ □
Where BC = □' B □□ and 'a' is constant of integration.
C
Again from Equations 8 and 9, we have
2A44 □ B44 □ C44 □ A4C4 □ A4B4 □ □16□p (18)
A B C AC AB
From Equations 12 and 18, we have
2A44 □ B44 □ C44 □ A4C4 □ A4B4 □ 0 (19)
A B C AC AB
Equation 19 gives
2 2
2(1□2n) □44 □(4n2 □2n□1) □□4 □ □ □□4 □ □0 (20)
□ □□ □ □□□□ □ □□
Where A = □n .

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From Equations 17 and 20, we have

2ff^1 □(4n^2 □2n□1) f^2 □a^2 □ (21)
(1□2n) □ □ (1□2n)□2n 1

Where □$_4$ = f(□).

Equation 21 leads to

f ^{2} □ 1 $_{2n}$ [a^2 □b(4n□1)□$^{4n□1/2n□1}$] (22)
(4n□1)□

Where 'b' is a constant of integration.

d□ (23)
log □ □ a 4n□^{1}□□ a2 □b(4n□1)□4n□1/2n□1

Hence, the metric of Equation 1 reduces to the form

ds^2 □ □ ^{d}f$^□$$_2$^{2} □□^{2n}dx^2 □□□dy^2 □□□dz^2 (24) Where □ is determined by Equation 23.

By introducing the following transformations

μ □ T, x□ X, y □ Y, z □ Z

Where

The metric of Equation 24 reduces to the form

dT ds2 □ □□ a2 □b□(4 4 n n □ □1 1)T T4^{2}n^n□1/2n□1 □□□ dT2 □T2ndX2 □T□dY2 □ T□ dZ2 (25) □

log □ □ a 4n□1□ T a2 □b(4n□1)T4n□1/2n□1 (26)

SOME PHYSICAL AND GEOMETRICAL FEATURES

The density for the model of Equation 25 is given by:

8 π□= (4n□1)b (27) 2(2n□1) $T^{\frac{4n^2+2n+1}{2n+1}}$

The tilt angle □ is given by:

coshλ = 1 2n □1 (28)
2 n

sinhλ = 1 1□2n (29)
2 n

The reality conditions

(i) □ + p > 0,

(ii) □ + 3p > 0, lead to

b(4n□1) $T^{\frac{-(4n^2+2n+1)}{(2n+1)}}$ □0 (30)
2(2n□1)

Where

b(4n□1)
□ 0
2(2n□1)

The scalar of expansion □ calculated for the flow vector □^{i} is given by:

(n□1) (2n□1)□a^2 □b(4n□1)T$^{4n□1/2n□1}$□ (31)
□□ n□1 (4n□1)Tn□1
2T

The components of fluid flow vector v^i and heat Bagora and Bagora 3 conduction vector q^i for the model of Equation 25 are given by:

$$□^1 □ \frac{1}{2T^n}\sqrt{\frac{1□2n}{n}} \quad (32)$$

$$□^4 □ \frac{1}{2}\sqrt{\frac{2n□1}{n}} \quad (33)$$

$$q^1 □ \frac{□(4n□1)b}{64□T^{\frac{6n^2□3n□1}{2n□1}}}\sqrt{\frac{1□2n}{n}} \quad (34)$$

4 (4n□1)(1□2n)b 1□2n (35) q □ □
n
64□(2n□1)T

The non-vanishing components of shear tensor ($□_{ij}$) and rotation tensor ($□_{ij}$) are given by

(4n^2 □1) (2n□1)□a^2 □b(4n□1)T$^{4n□1/2n□1}$□ (36)

□11 □ 24nT1□n n(4n□1)

□22 □12□Tnn((24nn□√□11)) □(1□2n) □a2 □b(4n□1)T4n□1/2n□1□□3a 4n□1□

(37)

□33 □ 121Tn n((24nn□√□11)) □(1□2n) □a2 □b(4n □1)T4n□1/2n□1□□3a 4n □1□ □

(38)

44 (2n □1)^2 (1□2n)□a^2 □b(4n □1)T$^{4n□1/2n□1}$□ (39)

□ □□ 24n n(4n □1)

□(26n^2 □3n □2) (1□2n)□a^2 □b(4n □1)T$^{4n□1/2n□1}$□

□14 □□

24nT n

(40)

ω_{14} = (6n□1) (1□2n)□a^2 □b(4n□1)T$^{4n□1/2n□1}$□ (41)

16T n(4n□1)

The rates of expansion H_i in the direction of x, y and z axes are given by

$$H_1 □ \frac{n}{T^{8n^2□4n□1/2(2n□1)}}\sqrt{\frac{a^2T^{4n^2□2n□1/(4n□1)}□b(4n□1)T^{2n}}{4n□1}} \quad (42)$$

$$H_2 □ \frac{1}{2T^{8n^2□4n□1/2(2n□1)}}\sqrt{\frac{a^2T^{4n^2□2n□1/(4n□1)}□b(4n□1)T^{2n}}{4n□1}} □ \frac{a}{2T^{n□1}} \quad (43)$$

$$H_3 □ \frac{1}{2T^{8n^2□4n□1/2(2n□1)}}\sqrt{\frac{a^2T^{4n^2□2n□1/(4n□1)}□b(4n□1)T^{2n}}{4n□1}} □ \frac{a}{2T^{n□1}} \quad (44)$$

DISCUSSION

The model started with a big-bang at T = 0 and the expansion in the model decrease as time T increases and it stopped at T = □. The model has point type singularity at T = 0 (MacCallum, 1971). The model represents shearing and rotating universe in general and rotation goes on decrease as time increases. Since Lim _□□0, then the model does not approach isotropy T □□ □ for large value of T. Density □ □ 0 as T □ □ and □ □ □ as T □ 0. When T □ 0. q^1 □ □ and q^4 □ □. Also, q^1 and q^4 tend to zero as T □ 0. At T = 0, the Hubble parameters tend to infinite at the time of initial singularity of vanish as T □ □.

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